Penrose 2020 Nobel Prize Proof is Unsound and Teleologically Biased

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1. Is Penrose's Paper to be Judged as Physics or Mathematics ?

The Royal Swedish Academy of Sciences, *Kungl. Vetenskaps Academien*, invites us to celebrate Dr. Roger Penrose's paper[1] on gravitational collapse of stars, and to view it in different ways — sometimes, as a theory of the physical world

- `` Roger Penrose showed that the general theory of relativity leads to the formation of black holes ´´ (Academien, 2020)
- `` Penrose's discovery triggered a new era in physics and astronomy. ´´ (Academien, 6 October 2020, pp. 7)

— and other times, as a mathematical proof

- `` Roger Penrose used ingenious mathematical methods in his proof ... ´´ (Academien, 2020)
- `` Penrose's result ... proves that gravitational collapse cannot be stopped after the trapped surface is formed ´´ (Academien, 6 October 2020, pp. 7)

1.1. Mathematics within Physics

Even if we were determined to regard Penrose's work as theoretical physics, we would then still be following in a cultural tradition where

> "... the theoretical physicist's picture of the world ... demands the highest possible standard of rigorous precision in the description of relations, such as only the use of mathematical language can give."(Einstein, 1954, pp. 225)

Thus taking Penrose's work as being dependent upon a mathematical description, we consider that

> "Since all arguments in pure mathematics are deductive, we can usually consider arguments that depend upon mathematics to be deductive as well.[2]"(Hurley, 2008, pp. 33-34)

Therein distinguishing *deductive* from *inductive*:

> "Deductive arguments are those in which the conclusion is claimed to follow necessarily from the premises, while inductive arguments are those in which the conclusion is claimed to follow only probably from the premises.[3] "(Hurley, 2008, pp. 72)

The rigorous standards of deductive argumentation that a pure mathematical proof is subject to, are not lessened for a mathematical description asserted within the context of physics.

1.2. Strategy of Mathematical Judgement first

Penrose's description will be judged here as a mathematical proof, evaluated as a deductive logic emancipated[4] from an experimental study of the physical world. This is also consistent with one strategy traditional within deductive argumentation most generally.

> "... if the premises fail to support the conclusion (that is, if the reasoning is bad) an argument is worthless. ... and only if the premises support the conclusion, will we test the factual claim"(Hurley, 2008, pp. 42)

[1] see my **§References**: (Penrose, 18 Jan 1965)

[2] "A noteworthy exception, however, is arguments that depend on statistics. ..., such arguments are usually best interpreted as inductive ´´(Hurley, 2008, pp. 34) — an exception not applicable here, as Penrose's paper neither offers nor references any statistics for his argument to depend upon.

[3] Notice that this definition of *inductive* is itself distinguishable from some traditional uses which indicate arguments that proceed from the particular to the general; see "... neither (like induction) from particular to general, nor (like syllogism) vice versa ..." (Aristotle, 1938b, pp. 516: note a)

This more ancient usage is not adopted as a limit here; rather "... there are deductive arguments that proceed from the general to the general, from the particular to the particular, and from the particular to the general, as well as from the general to the particular; and there are inductive arguments that do the same."(Hurley, 2008, pp. 37)

[4] "... a series of developments within mathematics and logic which have taken place ... one supreme result of this generalization has been the emancipation of logic and mathematics from the study of the real or the apparently real world."(Kuhn, 2023, pp. 117) And, in that sense, "Mathematics deals exclusively with the relations of concepts to each other without consideration of their relation to experience."(Einstein, 1950, pp. 41)

2. Key Symbols from Penrose's Argument

Most relevantly to deductive argumentation, I ask you to accept these following symbols from Penrose's paper, as labels for concepts that we can make exclusively **true**/**false** statements about.

C^3	T^2	$M_+{}^4$

Likewise, please accept Penrose's following labels for such statements about, in part, the relation of those concepts to each other.

(i)	(ii)	(iii)	(iv)	(v)

These labels having been foreshadowed to you, here's a top-down look at his argument.

3. On the Character of Penrose's Argument

The fundamental characteristic of Penrose's argument in (Penrose, 18 Jan 1965) is in it's negative conclusion. The last and conclusive sentence of his final logically substantive paragraph, para. 8, has the subject of "The impossibility ...". That sentence, like the rest of para. 8, seeks to fulfill a negative goal he set at the end of his para. 7:

> "It will be shown here, in outline, that (i), ... , (v) are together inconsistent."

Thereby, Penrose's argument proposes to expand *negative knowledge* — the knowledge of what is not true, rather than what is true.

3.1. At the Intersection of Falsehood and Equivocation

Consider assumptions "(i), ... , (v)" in his para. 7, grouped together in a boolean expression

$$(i)\cdot(ii)\cdot(iii)\cdot(iv)\cdot(v)$$

Expressing these assumptions as "together inconsistent" yields:

$$\overline{(i)\cdot(ii)\cdot(iii)\cdot(iv)\cdot(v)}$$

Therewith, his statement that "(i), ... , (v) are together inconsistent" translates algebraically as:

$$\overline{(i)\cdot(ii)\cdot(iii)\cdot(iv)\cdot(v)} \;= \textbf{true} \tag{1}$$

After negating both sides of equation (1), we will use the logically equivalent form:

$$(i)\cdot(ii)\cdot(iii)\cdot(iv)\cdot(v) \;= \textbf{false} \tag{2}$$

Therein, by rules of boolean algebra alone, one or more of these five assumptions must be false. Notice, however, that this formulation equivocates about which one or more assumptions are false.

To count all possible combinations of falsehood, consider the combinatorics of n assumptions chosen r at a time (satisfying $0 \le r \le n$):

$${}_nC_r = \frac{n!}{(n-r)! \times r!} \tag{3}$$

Thereby, the count for at least one (at a time) among the five assumptions being false is

$$\sum_{r=1}^{5} {}_5C_r \;= 31 \tag{4}$$

with each possible combination of falsehoods listed in the table immediately following.[5]

Combinatoric Domain of Falsehoods					
Grouping	(i)	(ii)	(iii)	(iv)	(v)
$_5C_1$	F	T	T	T	T
	T	F	T	T	T
	T	T	F	T	T
	T	T	T	F	T
	T	T	T	T	F
$_5C_2$	F	F	T	T	T
	F	T	F	T	T
	F	T	T	F	T
	F	T	T	T	F
	T	F	F	T	T
	T	F	T	F	T
	T	F	T	T	F
	T	T	F	F	T
	T	T	F	T	F
	T	T	T	F	F
$_5C_3$	F	F	F	T	T
	F	F	T	F	T
	F	F	T	T	F
	F	T	F	F	T
	F	T	F	T	F
	F	T	T	F	F
	T	F	F	F	T
	T	F	F	T	F
	T	F	T	F	F
	T	T	F	F	F
$_5C_4$	F	F	F	F	T
	F	F	F	T	F
	F	F	T	F	F
	F	T	F	F	F
	T	F	F	F	F
$_5C_5$	F	F	F	F	F

[5] This combinatoric approach does not account for dependancies among the assumptions, if any. In case of dependancies (e.g., if A is false, then B must be false), this table's violative rows are disqualified. The burden proof is on Penrose to limit combinations, whether or not for any dependency violations.

Penrose does not indicate which combination of falsehoods he intends. The mere assignment of the value **false** in equation (2) fails to fully engage the essentials of deductive argumentation, where truth and falsity are fundamental characteristics of statements. Statements (such as the premises he calls assumptions), when grouped together, have no boolean value attributed independently from that result computed by accounting for each value assigned to every premise in that group.

If we do not know each value the result is contingent upon, we cannot compute that result; more specifically, we cannot compute the left side of equation (2), to see whether it equals its right side.[6] Therein lies the fallacy[7] of *computational mysticism* — a fallacy that overlaps with Heisenberg's *equivocation*.[8] Thereby, Penrose leaves his readers open for guess-work about these details.

We could guess that Penrose, in assigning the value of false to the group, may have intended to imply that all premises in the group were false (as in the table's bottom row above, on level with ${}_5C_5$); however, I doubt it on grounds of deductive consequence. Consider, for example, his premise (v) :

> `` (v) There exists a trapped surface T^2 in $M_+{}^4$. ´´ (Penrose, 18 Jan 1965, pp. 59).

With this premise being false,

$$(v) \quad = \textbf{false} \tag{5}$$

this equation's semantics are that there does not exist a trapped surface T^2 in $M_+{}^4$ — semantics whose deductive consequence would call into question the Nobel Committee's appraisal:

> "The key concept that Penrose introduced was that of a *trapped surface* " (Academien, 6 October 2020, pp. 4: their italics)

Guess-work aside, Penrose's argument does assert that one or more of his premises must be false, even though we don't know which one(s). Considering that

> "A sound argument ... is what is meant by a "good" deductive argument in the fullest sense of the term."(Hurley, 2008, pp. 45)

our next question now becomes: Is Penrose's form of deductive argumentation a sound one?

3.2. An Unsound Deductive Argument

Consider that

> "A **sound argument** is a deductive argument that is *valid* and has *all true premises*. Both conditions must be met for an argument to be sound; if either is missing the argument is unsound. Thus, an **unsound argument** is a deductive argument that is invalid, has one or more false premises, or both."(Hurley, 2008, pp. 45: his italic and bold letters)

The second individually conclusive test for an unsound deductive argument is *one or more false premises*. With that test (and with my §3.1), a counter to Penrose's argument is:

- §3.1: Penrose's argument that "(i) ... (v) are together inconsistent" means that one or more premises "(i) ... (v)" must be false
- A deductive argument with one or more false premises is unsound

⇒ *therefore*, Penrose's deductive argument is unsound.

Despite Penrose's unsound deductive argument, let's see what aim he directs it towards.

3.3. Stated Aim of Penrose's Argument

Penrose's "The impossibility", his argument that "(i) ... (v) are together inconsistent", evokes in him his conception of *singularity* — a concept that ends his argument's contextual aim:

> "It will be shown that, after a certain critical condition has been fulfilled, deviations from spherical symmetry cannot prevent space-time singularities from arising."(Penrose, 18 Jan 1965, pp. 58: his underline)

[6] Admittedly, given the boolean operators in equation (2), once we know that one assumption is false, then that equation computes correctly regardless of each value associated with any other assumption; and then, each other value need not be accessed as a matter of computational efficiency.

Even so, the burden of proof is upon Penrose, to commit to which assumption is false. Furthermore, *not accessed* is not identical to *no value associated*.

[7] Following in the tradition that "A **fallacy** is a defect in an argument that consists in something other than false premises alone." (Hurley, 2008, pp. 113: his bold letters)

[8] Equivocation is a textbook fallacy in logical argumentation (Hurley, 2008, pp. 156-157 §3.19), but with a meaning distinguishable from my reading of Heisenberg. Despite following in Heisenberg's usage throughout this paper, I acknowledge that Hurley's textbook usage has historical roots reaching at least as far back as ancient Greece; see (Aristotle, 1938a, pp. 12-13).

Heisenberg's meaning is more expansive, requiring attention to specificity throughout, beyond Aristotle's attention to a double-meaning of words: "in a good ... scientific work, every detail must be laid down quite unequivocally; there can be no room for mere accident." (Heisenberg, 2007, pp. 18 in §P.S.).

What is this — *a certain critical condition*? The answer: a trapped surface T^2 has developed.[9]

So, returning us to the *initial* time in his model, he argues for two sequential stages to follow: first, that development of T^2; and thereafter,

> `` The argument will be to show that the existence of a trapped surface implies — irrespective of symmetry — that singularities necessarily develop. ´´
> — (Penrose, 18 Jan 1965, pp. 58)

Singularities are then shown to Penrose's satisfaction, as implied by logic of "The impossibility."

Whatever is apperceived by him in response to any fallacious logic, is superfluous here and hereafter. We now return to how he deduces the impossibility, regardless of what it evokes in him.

3.4. A Teleological Bias

In the body of his para. 8, Penrose argues why "(i), ... , (v) are together inconsistent." — his conclusive reason being an attribute of C^3 :

> "The impossibility ... follows from the non-compactness of C^3."

While Penrose explains *noncompact* elsewhere,[10] my emphasis right now is on his argument[11] that a logical contradiction within "(i), ... , (v)" follows from (an attribute of) C^3.

What do we know about C^3? From Penrose's last quote (with my own label prepended):

P_1 : C^3 is non-compact

Also, consider his other most-relevant premises (again, with my own labels prepended):

P_2 : C^3 exists
P_3 : C^3 is a Cauchy hyper-surface
P_4 : C^3 represents an initial matter distribution[12]

Possessing these premises about C^3, let's return to Penrose's "impossibility" — his proposal of negative knowledge about "(i), ... , (v)". After concluding why there is some inconsistency among them, he then stops his entire argument, as if having arrived at some fully-reasoned destination, or at least, his desired destination.

His abrupted argument begs the question:[13] how does he know that the fundamental issue is within (i), ... , (v)? Or, even if it is a fundamental issue, what would make us so sure that it is the only one ?

Looking at the same question-begging from a distinguishable perspective: if the logical contradiction involves C^3, then why isn't C^3 at issue? Or, at least, why isn't C^3 at issue among what Penrose has already cast aspersions at ?

In pursuit of that last possibility, let us suppose Penrose was correct (at least, in part) to cast aspersions in the direction he did. So, for the proverbial sake of argument, we suspend the critique in my §3.1, as we accept not first knowing which premises "(i), ... , (v)" are false; computationally, we now accept equation (2) as always correct. If so, it might compute correctly <u>because</u> C^3 is problematic — C^3 *is problematic* being our catch-phrase for the more formal

$$P_1 \cdot P_2 \cdot P_3 \cdot P_4 \quad = \textbf{false} \tag{6}$$

For example, consider his premise (*iii*):

> `` (*iii*) Every timelike or null geodesic in $M_+{}^4$ can be extended into the past until it meets C^3 (Cauchy hypersurface condition).
> ´´ (Penrose, 18 Jan 1965, pp. 58-59)

If C^3 is problematic because (at most, in part):

$$P_2 \quad = \textbf{false} \tag{7}$$

[9] As described within ``After the matter has contracted within $r = 2m$, a spacelike sphere S^2 (t = const, $2m > r$ = const) can be found in the empty region surrounding the matter. This sphere is an example of what will be called here a <u>trapped surface</u> — defined generally as a closed, spacelike, two-surface T^2 with the property that the two systems of null geodesics which meet T^2 orthogonally <u>converge</u> locally in future directions at T^2. ´´
— (Penrose, 18 Jan 1965, pp. 58: his underline)

[10] See (Penrose, Wed 22 May 2024, pp. 71-72), starting at his italicized *compact*.

[11] I do not review the entirety of his argument within his paragraph 8. Note that, in later years, he arrived at the same conclusion by taking advantage of work by Charles W. Misner — work which Penrose appraises as a simplification. See (Penrose, Wed 22 May 2024, pp. 71-72), starting at "... in my original paper I used a rather clumsy argument ...".

With his later work, as well, Penrose argues "But C^3 is non-compact, which immediately provides us with the required contradiction." (Penrose, Wed 22 May 2024, pp. 72)

[12] P_4 and P_3 re-worded from, and P_2 implied by: "Consider the time development of a Cauchy hypersurface C^3 representing an initial matter distribution."(Penrose, 18 Jan 1965, pp. 58)

[13] Begging the question is a textbook fallacy in logical argumentation (Hurley, 2008, pp. 149-152 §3.19)

then on that cause alone, equation (2) would compute correctly, as (*iii*) must be false: C^3 is not there for any geodesic extended into the past to meet, when C^3 does not exist — non-existence of C^3 being the semantics of equation (7).

Now, let us stop suspending any disbelief for the sake of argument. We return with something else Penrose said about how his concept ${M_+}^4$ relates to his concept C^3 .

> `` ... the manifold ${M_+}^4$, which is the future time development of an initial Cauchy hyper-surface C^3 (past boundary of the ${M_+}^4$ region) ... ´´ (Penrose, 18 Jan 1965, pp. 58)

Notice that both his last two quotes have something to do with *time*. Penrose uses temporal terms *past*, *future*, and *initial* — terms easily understood, at least in a folksy way.

Yet, this understanding is confounded by his mathematical representation of what he terms "the future". Read his last two quotes together, and see the reciprocally-linked circular relationship between his concepts ${M_+}^4$ and C^3 : the future development of C^3 is ${M_+}^4$ — ${M_+}^4$ being a representation with an attribute ("in ${M_+}^4$") which extends from that future into its past to meet with the initial representation C^3 whose future development is ${M_+}^4$.

Penrose is free to label this reciprocating logic anything he wants, including "the future"; but that academic freedom does not interrupt his logical circularity which confounds *the future* with *the initial*. This circularity impinges upon our earlier discussion of "(i), ... , (v)":

- Premises "(*i*) ... (*v*)" each relates to ${M_+}^4$ (see his para. 7, "(*i*) ... (*v*)", to corroborate).
- A logically reciprocal relationship exists between the concepts ${M_+}^4$ and C^3

$\Rightarrow$ *therefore*, "(*i*) ... (*v*)" each relates to C^3.

So, inconsistencies among "(i), ... , (v)" each relate to C^3 ; given this, I direct our attention to my under-specifying the full combinatorics of falsehood, earlier in my §3.1 . Rather than using only those five assumptions in equation (2), the now-relevant premises are these nine within

$$(i)\cdot(ii)\cdot(iii)\cdot(iv)\cdot(v)\cdot \mathrm{P}_1\cdot \mathrm{P}_2\cdot \mathrm{P}_3\cdot \mathrm{P}_4 = \textbf{false} \quad (8)$$

Admittedly, I would inflate the domain over which Penrose equivocates, if counting

$$\sum_{r=1}^{9} {}_9C_r \quad = 511 \quad (9)$$

as this does not discount for dependancies among the assumptions; e.g., we already know from earlier in this §3.4, that (*iii*) must be false when P_2 is false. All combinations violative of any dependency do not count. Yet, remaining combinations that match (in part):

$$\mathrm{P}_2 = \textbf{true} \quad (10a)$$
$$\mathrm{P}_3 = \textbf{false} \quad (10b)$$
$$\mathrm{P}_4 = \textbf{true} \quad (10c)$$

indicate that an initial representation (named C^3) exists, but that the representation is not a Cauchy hyper-surface. Similarly, all which match:

$$\mathrm{P}_2 = \textbf{true} \quad (11a)$$
$$\mathrm{P}_3 = \textbf{true} \quad (11b)$$
$$\mathrm{P}_4 = \textbf{false} \quad (11c)$$

indicate that an extant Cauchy hyper-surface C^3 falsely represents an initial matter distribution.

With such combinations, Penrose could have used his same unsound deductive technique towards his same assumptions, within a demonstration about some difficulties of using a Cauchy hyper-surface as an initial representation. Instead, his stated aim is biased towards the contradictions of his logic to become manifest later in time.[14]

We could say that his premises support his conclusion, in that biased sense; and, also in the fallacious sense that they support another — the conclusion that his initial representation makes his argument wrong-headed from the beginning.

4. Conclusion

Penrose argues in favor of an impossibility. Notice, to clarify my counter-argument by contrast, that I do not object to Penrose's argument merely because it proposes a negative knowledge. Rather, as I now emphasize, knowledge of what is not true has sometimes been useful within physics very generally; for example,

> "... There is no absolute motion.
>
> It might seem that our insight would gain little from such a negative statement. In reality, however, it is a strong restriction for the conceivable laws of nature. "(Einstein, 1950, pp. 41)

Also, papers before Penrose's have proposed negative knowledge about gravitational collapse; e.g.,

[14] Or, if you accept an interpretation by the Nobel committee — a bias towards when "Time ends at the singularity." (Academien, 6 October 2020, pp. 5: Fig. 1 caption)

> " ... the question, answered by this paper in the negative, as to whether physical models are capable of exhibiting such a [Schwarzschild] singularity."(Einstein, October 1939, pp. 936: my bracketed edit)

The counter-argument I developed in this paper does indeed object, rather, to the relationship between his conclusion and his premises. They conflict — the exact contradiction, however, being obscured by his equivocation.

Logical contradiction between two statements is not absolved by one statement being a premise, and the other a conclusion. To set up a simplified example of that fault, first consider a valid syllogism widespread in academia:[15]

- Socrates is a man
- All men are mortal

⇒ *therefore*, Socrates is mortal.

When the conclusion negates a premise, we get:

- Socrates is a man
- All men are mortal

⇒ *therefore*, All men are not mortal

or, ⇒ *therefore*, Socrates is not a man.

Neither contradiction is resolved, when we equivocate which conclusion we are committing to, nor even when we're committing to both conclusions.

These principles (scaled-up from only two premises to a different nine from Penrose) inform my counter-argument, where I critique the source of Penrose's equivocation, and I introduce further limitations to that finite domain over which he equivocates.

Any critique, if discerning, is the shadow-side of standards held more highly. The highest standard of deductive argumentation seeks:

- reasons or evidence

and • an inferential claim such that the stated conclusion follows necessarily, when those reasons or evidence are actually true.

Determining the validity[16] of an inferential claim is, for all but straight-forward syllogisms, a generally difficult challenge in logic.

> "Nevertheless, there is *one* arrangement of truth and falsity in the premises and conclusion that does determine the issue of validity. Any deductive argument having actually true premises and an actually false conclusion is invalid."(Hurley, 2008, pp. 44 : his italics)

This most important point in all of deductive logic[17] provides a rigorously reliable negative feedback about an inferential claim — a feedback uniquely available to those who pursue new knowledge from premises of actual truth.

So, even when Einstein proposed a negative knowledge in his paper quoted earlier, his premises were (to his best knowledge) actually true; consider Einstein's compound statement:

> "... matter cannot be concentrated arbitrarily. And this is due to the fact that otherwise the constituting particles would reach the velocity of light. "(Einstein, October 1939, pp. 936)

Admittedly, in the future, somebody may indeed be the first to observe particles reaching the velocity of light. But even if Einstein's speed limitation were actually true, that would admittedly be no necessary indication that his argument was the wisest argument possible. Other inferential claims with distinguishable premises or conclusions can always compete.[18]

Notwithstanding these indications that no human has a God-like relation to the Truth, Einstein's deductive argument dealt in premises stated clearly without equivocation, that he forthrightly assumed were true.

Thereby and in the wider context of physics where predictions concluded logically are expected to be tested by experiment, he risked his inferential claim to rigorously reliable feedback. In this way, Einstein's paper conforms to a higher standard of deductive argumentation, than Penrose's paper does.

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[15] For one instance, see (Kuhn, 2023, pp. 119)

[16] See (Hurley, 2008, pp. 42-45 §1.4) in context of (Hurley, 2008, pp. 14-15 §1.2)

[17] Variant of "The idea that any deductive argument having actually true premises and a false conclusion is invalid may be the most important point in all of deductive logic." (Hurley, 2008, pp. 44), strengthened by dropping his *may*(-be) qualifier on *the most important point*.

He and I both agree that "The entire system of deductive logic would be quite useless if it accepted as valid any inferential process by which a person could start with truth in premises and arrive at a falsity in the conclusion."(Hurley, 2008, pp. 44)

[18] "... by a suitable coordinate choice you can — somewhat remarkably — get rid of this 'Schwarzschild Singularity'"(Penrose, Wed 22 May 2024, pp. 63)

References

Aristotle, translation and commentary by Harold P. Cooke, “The Categories on Interpretation” in *The Organon,* pp. 1-182, Harvard University Press, Cambridge, Massachusetts (1938).

Aristotle, translation and commentary by Hugh Tredennick, “Prior Analytics” in *The Organon,* pp. 183-532, Harvard University Press, Cambridge, Massachusetts (1938).

Albert Einstein, *Out of My Later Years,* Philosophical Library Inc., New York (1950).

Albert Einstein, *Ideas and Opinions,* Crown Publishers, New York (1954).

Albert Einstein, “ON A STATIONARY SYSTEM WITH SPHERICAL SYMMETRY CONSISTING OF MANY GRAVITATING MASSES,” *Annals of Mathematics,* Second Series **40**(4), pp. 922-936 (October 1939).

Werner Heisenberg, *Physics and Philosophy,* HarperCollins Publishers, New York (2007). First published (without §P.S.) in 1958, by Harper & Brothers, New York.

Patrick J. Hurley, *A Concise Introduction to Logic,* Wadsworth, Belmont, CA (2008). Tenth Edition.

Thomas S. Kuhn, *The Quest for Physical Theory,* MIT Libraries Department of Distinctive Collections (2023). Thomas S. Kuhn’s Lowell Lectures Delivered in March 1951.

Kungl Vetenskaps Academien, *Press release: The Nobel Prize in Physics* (2020). Available at: https://www.nobelprize.org/prizes/physics/2020/press-release/ .

Kungl Vetenskaps Academien, The Nobel Committee for Physics, *Scientific Background on the Nobel Prize in Physics 2020* (6 October 2020). at: url:https://www.nobelprize.org/uploads/2020/10/advanced-physicsprize2020.pdf .

Roger Penrose, “GRAVITATION COLLAPSE AND TIME-SPACE SINGULARITIES,” *Physical Review Letters* **14**(3), pp. 57-59, American Physical Society (18 Jan 1965). doi 10.1103/PhysRevLett.14.57 url:https://link.aps.org/doi/10.1103/PhysRevLett.14.57.

Roger Penrose, *Nobel Lecture,* NobelPrize.org. (Wed 22 May 2024). Nobel Prize Outreach AB 2024. <https://www.nobelprize.org/prizes/physics/2020/penrose/lecture/>.